

Orientational glass: full replica symmetry breaking in generalized spin glass-like models without reflection symmetry

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Abstract

We investigate near the point of glass transition the expansion of the free energy corresponding to the generalized Sherrington–Kirkpatrick model with arbitrary diagonal operators \hat{U} standing instead of Ising spins. We focus on the case when \hat{U} is an operator with broken reflection symmetry. Such a consideration is important for understanding the behavior of spin-glass-like phases in a number of real physical systems, mainly in orientational glasses in mixed molecular crystals which present just the case. We build explicitly a full replica symmetry breaking (FRSB) solution of the equations for the orientational glass order parameters when the non-symmetric part of \hat{U} is small. This particular result presents a counterexample in the context of usually adopted conjecture of the absence of FRSB solution in systems with no reflection symmetry.

I. INTRODUCTION

The theory of spin glasses has been developed as an attempt to describe unordered equilibrium freezing of spins in actual dilute magnetic systems with disorder and frustration. This problem was soon solved at the mean-field level [1–5] [see also Ref. [6] for a review]. Below the Almeida – Thouless line [3] the replica symmetric solution was shown to be incorrect. Parisi proposed the method of replica symmetry breaking (RSB) step by step with the limit — full RSB (FRSB) when glass order parameter becomes a continuous non-decreasing function $q(x)$ of a parameter $0 \leq x \leq 1$. It provides the hierarchical distribution of pure states overlaps probability $P(q)$ through $P(q) = dx/dq$. This approach allows to describe the main features of the experiments on spin glasses.

A number of generalizations of the Sherrington and Kirkpatrick (SK) model with Ising spins have been considered. But still the problem how RSB and FRSB solutions do occur remains relevant and far from being completely understood [7–13]. It was shown that the violation of the replica symmetry is correlated with the symmetry properties of the Hamiltonian [14–19]. There is a conjecture that in the absence of the reflection symmetry it is not possible to construct a continuous non-decreasing function $q(x)$ and so, the FRSB solution does not exist in this case, or at least does not occur in the point of RS solution instability. It is possible that in some of these models different stages of replica symmetry breaking occur not in the first symmetry breaking point as in the SK model (see, for example, Refs. [20, 21], where the Potts model is considered).

In the literature the absence of reflection (or time reversal) symmetry usually was incorporated in the structure of the Hamiltonian. It is so, for example, for Potts spin glasses and for p -spin spin glass with the interaction of p Ising spins. However, there is another way to break reflection symmetry – that is to incorporate the breaking in the character of the operators themselves while the structure of the Hamiltonian remains two-particle as in the SK model. As far as we know the present paper is the first one to deal with RSB in such a situation.

It is worth to notice that, while, say, p -spin model has a physical meaning only as a prototype of structural glasses, there exists a number of real physical systems which can be described in terms of two-operator random systems [17, 22, 23] of the SK type where the absence of reflection symmetry is caused by the characteristics of the operators U themselves.

Let us list some physical examples of such systems. They are just described by the Hamiltonians of the SK type but with Ising spins changed for non reflection symmetrical operators \hat{U} . The investigation of replica symmetry breaking presented in this paper can give some new information about low-temperature behavior of these systems.

For example ortho – para–hydrogen mixed crystals and in $Ar - N_2$ present mixtures of spherically symmetric molecules and momentum–bearing molecules. The corresponding orientational quadrupolar glass was investigated on the base of the S-K type Hamiltonian with $U = Q$, where $Q = 3J_z^2 - 2$, $\mathbf{J} = 1$ [24]. Another example of a SG–like phase in molecular crystal is presented by pure para – H_2 (or ortho – D_2) under pressure. The possibility of orientational order in systems of initially spherically symmetric molecule states is due to the involving of higher order orbital moments $J = 2, 4, \dots$ in the physics under pressure. The frustration and disorder give the basis to the investigation of quadrupole glass with $J = 2$. Now $\hat{U} = (1/3)(3J_z^2 - 6)$. Such a theory was constructed in Ref. 22. Another interesting example of spin-glass-like phase is presented by the orientational glass that appears in molecular C_{60} at $90K$. Now the sample is not a mixture and the form of the molecules is almost spherical. Nevertheless the existing anisotropy of the potential causes a frustration. The dependence of the anisotropic part of the potential on mutual orientation of molecules has two pronounced minima and so the effect of a mixture is obtained. This is the base of the theory with \hat{U} being a superposition of cubical harmonics [25].

The aim of this work – to find out for which class of \hat{U} -operators that do not have the reflection symmetry the problem can be described by the Parisi FRSB scheme. In this paper, we do the first step in this direction. In Ref. [26] it was shown that in the case of arbitrary \hat{U} -operators satisfying the condition of the reflection symmetry there is full replica symmetry breaking. Keeping in mind that the FRSB solution is valid for operators with the reflection symmetry we seek for the FRSB solution for operators \hat{U} represented by the sum of a symmetric operator and a small perturbation that has no reflection symmetry. This investigation is close in spirit to the problem of building the FRSB solution for the SK model in a weak field [5].

II. THE MODEL

We start with the Hamiltonian

$$H = -\frac{1}{2} \sum_{i \neq j} J_{ij} \hat{U}_i \hat{U}_j. \quad (1)$$

where arbitrary diagonal operators \hat{U} are located on the lattice sites i . The quenched interactions J_{ij} are distributed with the Gaussian probability

$$P(J_{i,j}) = \frac{\sqrt{N}}{\sqrt{2\pi}J} \exp \left[-\frac{(J_{i,j})^2 N}{2J^2} \right], \quad (2)$$

where N is the number of sites. Using replica approach we can write in standard way the free energy averaged over disorder in the form

$$\langle F \rangle_J / NkT = \lim_{n \rightarrow 0} \frac{1}{n} \max \left\{ \frac{t^2}{4} \sum_{\alpha} (w^{\alpha})^2 + \frac{t^2}{2} \sum_{\alpha > \beta} (q^{\alpha\beta})^2 - \ln \text{Tr}_{\{U^{\alpha}\}} \exp \hat{\theta} \right\}. \quad (3)$$

Here

$$\hat{\theta} = t^2 \sum_{\alpha > \beta} (q^{\alpha\beta}) \hat{U}^{\alpha} \hat{U}^{\beta} + \frac{t^2}{2} \sum_{\alpha} (w^{\alpha}) (\hat{U}^{\alpha})^2, \quad (4)$$

where $t = J/kT$. The saddle point conditions were used to define the glass order parameter

$$q^{\alpha\beta} = \text{Tr} \left[\hat{U}^{\alpha} \hat{U}^{\beta} \exp \left(\hat{\theta} \right) \right] / \text{Tr} \left[\exp \left(\hat{\theta} \right) \right], \quad (5)$$

and the auxiliary order parameter

$$w^{\alpha} = \text{Tr} \left[(\hat{U}^{\alpha})^2 \exp \left(\hat{\theta} \right) \right] / \text{Tr} \left[\exp \left(\hat{\theta} \right) \right]. \quad (6)$$

In the RS approximation from the extremum condition for the free energy for the glass order parameter q_{RS} we have:

$$q_{\text{RS}} = \int dz^G \left\{ \frac{\text{Tr} \left[\hat{U} \exp \left(\hat{\theta}_{\text{RS}} \right) \right]}{\text{Tr} \left[\exp \left(\hat{\theta}_{\text{RS}} \right) \right]} \right\}^2. \quad (7)$$

and for the auxiliary order parameter:

$$w_{\text{RS}} = \int dz^G \frac{\text{Tr} \left[\hat{U}^2 \exp \left(\hat{\theta}_{\text{RS}} \right) \right]}{\text{Tr} \left[\exp \left(\hat{\theta}_{\text{RS}} \right) \right]}. \quad (8)$$

Here

$$\hat{\theta}_{\text{RS}} = zt\sqrt{q_{\text{RS}}}\hat{U} + t^2\frac{[w_{\text{RS}} - q_{\text{RS}}]}{2}\hat{U}^2 \quad (9)$$

and

$$\int dz^G = \int_{-\infty}^{\infty} \frac{dz}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right). \quad (10)$$

The stability of the RS solution can be tested by the investigation of the gaussian fluctuations contribution to the free energy near this solution. The solution is stable while all the eigen modes of the fluctuation propagator are positive. The most important mode is the so-called replicone mode [3, 27] since only its sign is usually sensitive to the replica symmetry breaking degree and to the temperature. For example, the replica symmetric solution is stable unless the corresponding replicon mode energy $\lambda_{(\text{RS})\text{ repl}} > 0$. The RS-solution can break at the temperature T_c determined by the equation $\lambda_{(\text{RS})\text{ repl}} = 0$, where

$$\lambda_{(\text{RS})\text{ repl}} = 1 - t^2 \int dz^G \left\{ \frac{\text{Tr}(\hat{U}^2 e^{\hat{\theta}_{\text{RS}}})}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} - \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right]^2 \right\}^2. \quad (11)$$

The equation $\lambda_{(\text{RS})\text{ repl}} = 0$ is nothing else that the bifurcation condition for 1RSB equation for glass order parameter [23, 29], i.e., as the condition that a small solution with 1RSB can appear. Analogously the other $\lambda_{(n-1)\text{RSB}}$ and the bifurcation condition at the n -th stages of the replica symmetry breaking are related.

We break the RS once again and obtain the corresponding expressions for the free energy and the order parameters. The bifurcation condition $\lambda_{(\text{1RSB})\text{ repl}} = 0$ determining the temperature follows from the condition that a nontrivial small solution for the 2RSB glass order parameter appears. [23, 27, 29] We have:

$$\lambda_{(\text{1RSB})\text{ repl}} = 1 - t^2 \int dz^G \frac{\int ds^G \left[\text{Tr} \exp \hat{\theta}_{\text{1RSB}} \right]^m \left\{ \frac{\text{Tr}[\hat{U}^2 \exp \hat{\theta}_{\text{1RSB}}]}{\text{Tr}[\exp \hat{\theta}_{\text{1RSB}}]} - \left[\frac{\text{Tr}[\hat{U} \exp \hat{\theta}_{\text{1RSB}}]}{\text{Tr}[\exp \hat{\theta}_{\text{1RSB}}]} \right]^2 \right\}^2}{\int ds^G \left[\text{Tr} \exp \hat{\theta}_{\text{1RSB}} \right]^m}, \quad (12)$$

where θ_{1RSB} is the analog of (9) for the first stage of RSB and m - 1RSB order parameter.

III. FRSB SOLUTION

Let us consider first a generalized model defined by the Hamiltonian (1) with reflection symmetrical operators U . The reflection symmetry implies that for any integer k ,

$$\text{Tr} \left[\hat{U}^{(2k+1)} \right] = 0. \quad (13)$$

In the RS-approximation we find the solution q_{RS} that is zero at high temperature. The bifurcation condition in this case is:

$$1 - t_c^2 w_{\text{RS}}^2(t_c) = 0. \quad (14)$$

This equation coincides with $\lambda_{(\text{RS})\text{ repl}} = 0$ [see, e.g., Ref. 6]. It is zero high temperature solution that bifurcates. At $T < T_c$ certain nontrivial 1RSB solutions appear but they are unstable.

Investigating 1RSB, 2RSB, 3RSB, ..., n RSB, and so on, we see that the equations for the glass order parameters always contain the quantity

$$\text{Tr}[U \exp(\theta_{n\text{RSB}})] / \text{Tr}[\exp(\theta_{n\text{RSB}})]. \quad (15)$$

Here $\theta_{n\text{RSB}}$ are the analogs of (9) for higher stages of RSB (see Ref. 26 for details). Therefore, one of the solutions of this equation is trivial at each RSB-step, and the appearance of the n RSB solution can be regarded as the bifurcation of the trivial $(n - 1)$ RSB solution. In this case, the equation $\lambda_{n\text{RSB}} = 0$ coincides with the corresponding branching condition (14). This means that in any case, the n RSB solutions at different stages of the symmetry breaking can exist at temperature $T < T_c$ determined by this bifurcation condition, and so we always can look for FRSB solution. Writing the free energy as a series over $\delta q^{\alpha\beta}$ near T_c (up to the fourth order) we obtain $q(x) = cx$ in the leading approximation [a similar procedure was described in details in Ref. 26].

If the operators \hat{U} do not have the reflection symmetry, $\text{Tr} \hat{U}^{(2k+1)} \neq 0$, then the glass freezing scenario is different from the previous case. The characteristic properties of system develop themselves already in the replica symmetry approximation. The nonlinear integral equation for the RS-glass order parameter simply has no trivial solutions at any temperature because the integrand is nonsymmetric due to the cubic terms in the free-energy expansion [23, 27]. There is a smooth increase in the RS order parameters as the temperature decreases. The bifurcation condition $\lambda_{(\text{RS})\text{ repl}} = 0$ (11) defines the point T_c where the RS-solution becomes unstable.

Similarly to the previous case, considering respectively 1RSB, 2RSB, 3RSB, ..., n RSB, we find that the equation $\lambda_{n\text{RSB}} = 0$ always has the solution which determines the point T_c and coincides with the solution of equation $\lambda_{(\text{RS})\text{ repl}} = 0$ [23, 27, 29]. It is important that it is the non-zero solution that bifurcates.

Looking now in general at the free-energy series over the glass order parameter we see that the series contain explicitly the terms which can be classified by the reflection symmetry.

To estimate the form of the FRSB-solution near the bifurcation point T_c at which it ceases to coincide with the RS-solution (i.e., in neighborhood of T_c), we expand the expression for the free energy up to the fourth order (inclusively), assuming that the deviations $\delta q^{\alpha\beta}$ from q_{RS} are small. We believe that one can neglect the changes of the order parameter w_{RS} (see Ref. 29 where it is directly shown for 1RSB). We obtain the deviation ΔF of the free energy from its RS part:

$$\begin{aligned} \frac{\Delta F}{NkT} = \lim_{n \rightarrow 0} \frac{1}{n} & \left\{ \frac{t^2}{4} [1 - t^2 W] \sum'_{\alpha, \beta} (\delta q^{\alpha\beta})^2 - \frac{t^4}{2} L \sum'_{\alpha, \beta, \delta} \delta q^{\alpha\beta} \delta q^{\alpha\delta} - t^6 \left[B_2 \sum'_{\alpha, \beta, \gamma, \delta} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\beta\delta} + \right. \right. \\ & B'_2 \sum'_{\alpha, \beta, \gamma, \delta} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\alpha\delta} + B_3 \sum'_{\alpha, \beta, \gamma} \delta q^{\alpha\beta} \delta q^{\beta\gamma} \delta q^{\gamma\alpha} + B'_3 \sum'_{\alpha, \beta, \gamma} (\delta q^{\alpha\beta})^2 \delta q^{\alpha\gamma} + B_4 \sum'_{\alpha, \beta} (\delta q^{\alpha\beta})^3 \Big] + \\ & t^8 \left[D_2 \sum'_{\alpha, \beta} (\delta q^{\alpha\beta})^4 + D_{31} \sum'_{\alpha, \beta, \gamma} (\delta q^{\alpha\beta})^3 \delta q^{\alpha\gamma} + D_{32} \sum'_{\alpha, \beta, \delta} (\delta q^{\alpha\beta})^2 (\delta q^{\alpha\delta})^2 + D_{33} \sum'_{\alpha, \beta, \gamma} (\delta q^{\alpha\beta})^2 \delta q^{\alpha\gamma} \delta q^{\gamma\beta} + \right. \\ & D_{42} \sum'_{\alpha, \beta, \gamma, \delta} (\delta q^{\alpha\beta})^2 \delta q^{\alpha\gamma} \delta q^{\alpha\delta} + D_{43} \sum'_{\alpha, \beta, \gamma, \delta} (\delta q^{\alpha\beta})^2 \delta q^{\alpha\gamma} \delta q^{\beta\delta} + D_{45} \sum'_{\alpha, \beta, \gamma, \delta} (\delta q^{\alpha\beta})^2 \delta q^{\alpha\gamma} \delta q^{\gamma\delta} + \\ & D_{46} \sum'_{\alpha, \beta, \gamma, \delta} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\alpha\delta} \delta q^{\beta\gamma} + D_{47} \sum'_{\alpha, \beta, \gamma, \delta} \delta q^{\alpha\beta} \delta q^{\beta\gamma} \delta q^{\gamma\delta} \delta q^{\delta\alpha} + D_{53} \sum'_{\alpha, \beta, \gamma, \delta, \mu} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\alpha\delta} q^{\gamma\mu} + \\ & \left. D_{54} \sum'_{\alpha, \beta, \gamma, \delta, \mu} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\alpha\delta} q^{\beta\mu} + D_{55} \sum'_{\alpha, \beta, \gamma, \delta, \mu} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\gamma\delta} q^{\delta\mu} \right] \Big\}, \quad (16) \end{aligned}$$

where $t = t_c + \Delta t$. The prime on the sum means that only the superscripts belonging to the same δq are necessarily different in \sum' . The expressions for coefficients W, L, \dots, D are given in Appendix. All coefficients depend only on RS-solution at T_c . Note that $1 - t^2 W = \lambda_{(\text{RS}) \text{ repl}}$. The expression (16) includes a part without the reflection symmetry, namely, the terms with odd number of identical replica indices (see also [11–13]).

To write the free energy as a functional of $q(x)$ we use the standard formalized algebra rules [5, 6]. The properties of this algebra were formulated by Parisi for Ising spin glasses. In our case, the expansion of the generalized expression for the free energy Eq.(16) includes some terms of non-standard form. Those terms are not formally described by the Parisi rules, but can be easily reduced to the standard form. To do this, we compared the corresponding expression, consistently producing 1RSB, 2RSB, ... symmetry breaking. For example,

$$\lim_{n \rightarrow 0} \frac{1}{n} \sum'_{\alpha, \beta, \gamma, \delta} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\alpha\delta} = \lim_{n \rightarrow 0} \frac{1}{n} \sum'_{\alpha, \beta, \gamma, \delta} \delta q^{\alpha\beta} \delta q^{\alpha\gamma} \delta q^{\beta\delta} \quad (17)$$

The equation for order parameter follows from the stationarity condition $(\delta/\delta q(x))\Delta F = 0$ applied to the free energy functional. The resulting complicated integral stationarity equation can be simplified using the differential operator $\hat{O} = \frac{1}{q'} \frac{d}{dx} \frac{1}{q'} \frac{d}{dx}$, where $q' = \frac{dq(x)}{dx}$. As a result, we obtain:

$$t^6 \{B_4 - B_3 x\} + t^8 \left\{ -D_{46} x \langle q \rangle + D_{47} \left[-4q(x)x^2 - 4x \langle q \rangle + 4x \int_0^x dy q(y) \right] + D_{31} \langle q \rangle + D_{33} \left[4q(x)x + 2 \langle q \rangle - 2 \int_0^x dy q(y) \right] - 4D_2 q(x) \right\} = 0, \quad (18)$$

where $\langle q \rangle = \int_0^1 dy q(y)$. Our results agree with those obtained in Ref. 28 in the case when \hat{U} are Ising spins and $(\hat{U})^2 = 1$.

Differentiating the equation Eq.(18) we obtain in the leading approximation:

$$q' = \frac{B_3}{t^2 4 [-D_2 + D_{33}x - D_{47}x^2]}. \quad (19)$$

In deriving the equation Eq.(19) we have neglected in the numerator of the members terms of the form $q(x) \sim \tau = (T_c - T)/T_c$ compared with the constant B_3 . This is true for the consideration of Parisi [4–6]. Since the model with the operators \hat{U} with reflective symmetry (i.e. $\text{Tr} [\hat{U}^{(2k+1)}] = 0$) exactly the same as the model of the Ising spins [26], for reasons of continuity, we consider operators who have a very small $\text{Tr} [\hat{U}^{(2k+1)}]$.

It is easy to see that

$$B_3 = \frac{1}{6} \int dz^G \left\{ \frac{\text{Tr} (\hat{U}^2 e^{\hat{\theta}_{\text{RS}}})}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} - \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right]^2 \right\}^3 \geq 0. \quad (20)$$

It follows from the Cauchy–Schwarz inequality that the expression $\left(\text{Tr} \hat{U}^2 e^{\hat{\theta}_{\text{RS}}} \right) \left(\text{Tr} e^{\hat{\theta}_{\text{RS}}} \right) \geq \left(\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}} \right)^2$ follows from $(\sum_n A_n^2) (\sum_n B_n^2) \geq (\sum_n A_n B_n)^2$.

We have also

$$\begin{aligned}
-D_2 = \frac{1}{48} \int dz^G & \left\{ \frac{\text{Tr}(\hat{U}^4 e^{\hat{\theta}_{\text{RS}}})}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} - 3 \left[\frac{\text{Tr} \hat{U}^2 e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right]^2 - \right. \\
& 6 \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right]^4 + 12 \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right]^2 \left[\frac{\text{Tr} \hat{U}^2 e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right] - \\
& \left. 4 \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right] \left[\frac{\text{Tr} \hat{U}^3 e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right] \right\}^2 \geq 0. \quad (21)
\end{aligned}$$

$$-D_{47} = \frac{1}{8} \int dz^G \left\{ \frac{\text{Tr}(\hat{U}^2 e^{\hat{\theta}_{\text{RS}}})}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} - \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right]^2 \right\}^4 \geq 0. \quad (22)$$

If \hat{U} are operators with reflection symmetry, then $D_{33} = 0$, and the denominator of (18) is positive. So, in the case of reflection symmetry $q' > 0$.

If the operators \hat{U} have no reflection symmetry, then $D_{33} \neq 0$ and the denominator of (18) can be negative. However, for reasons of continuity, one can imagine that it is not always the case. As an example, let us consider the operators $\hat{U} = \hat{S} + \eta \hat{Q}$ where η is small. Here \hat{S} is the z -component of spin (for $\mathbf{S} = 1$) taking values $(0, 1, -1)$. While \hat{Q} is the axial quadrupole moment, $\hat{Q} = 3\hat{S}^2 - 2$, and it takes values $(-2, 1, 1)$ (see, e.g. [26]). The operators \hat{Q} and \hat{S} have the following properties: $\hat{Q}^2 = 2 - \hat{Q}$, $3\hat{S}^2 = 2 + \hat{Q}$, and $\hat{Q}\hat{S} = \hat{S}\hat{Q} = \hat{S}$. So the algebra of these operators is closed. The operator \hat{S} has the reflection symmetry while \hat{Q} has not. FRSB is valid for arbitrary reflection symmetric operators [26], in particular, for \hat{S} . Let us note that the operator $\sqrt{3}S = V$ is a second component of the quadrupole momentum operator $V = S_x^2 - S_y^2$ considered in the problem of anisotropic quadrupolar glass.

The detailed calculation was performed and it was shown that for small η there is the vicinity of T_c where $q'(x)$ remains to be positive. The area where $q(x)$ depends on x is small and the function $q(x)$ is small $\sim \tau = (T_c - T)/T_c$.

Thus we can construct correct FRSB solution with $q' > 0$ in the absence of reflection symmetry of operators \hat{U} .

It is a possibility that at lower temperature the D_{33} term occurs to be larger so that the derivative (18) becomes negative and the FRSB ceases to exist. This fact can give rise to a “reverse” behavior of the system as compared with “standard” case proposed in Ref. 20 for Potts glass models.

IV. CONCLUSIONS

So, we have considered a model with two-particle interaction where the absence of reflection symmetry is caused by the characteristics of the operators U themselves. FRSB is first described in such a system. An expansion for the free energy of our generalized SK model with arbitrary operators \hat{U} standing instead of Ising spins is investigated near the RSB transition point T_c . The principal prescription for obtaining a full replica symmetry breaking solution is derived in general case. In a case when \hat{U} is a reflection symmetric operator with a nonsymmetric perturbative part the FRSB solution is constructed explicitly.

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VI. APPENDIX

$$W = \langle \hat{U}_1^2 \hat{U}_2^2 \rangle - 2 \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \rangle + \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \rangle = \int dz^G \left\{ \frac{\text{Tr} \left(\hat{U}^2 e^{\hat{\theta}_{\text{RS}}} \right)}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} - \left[\frac{\text{Tr} \hat{U} e^{\hat{\theta}_{\text{RS}}}}{\text{Tr} e^{\hat{\theta}_{\text{RS}}}} \right]^2 \right\}^2. \quad (23)$$

Notation used below are obvious from the equation (23). The coefficients of the terms of the third and the fourth order we write out only those that are included in the final equation.

$$L = \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \rangle - \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \rangle. \quad (24)$$

$$B_4 = \frac{1}{3} \langle \hat{U}_1 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \hat{U}_6 \rangle - \langle \hat{U}_1^2 \hat{U}_2 \hat{U}_3 \hat{U}_4 \hat{U}_5 \rangle + \frac{1}{3} \langle \hat{U}_1^3 \hat{U}_2 \hat{U}_3 \hat{U}_4 \rangle + \frac{3}{4} \langle \hat{U}_1^2 \hat{U}_2^2 \hat{U}_3 \hat{U}_4 \rangle - \frac{1}{2} \langle \hat{U}_1^3 \hat{U}_2^2 \hat{U}_3 \rangle + \frac{1}{12} \langle \hat{U}_1^3 \hat{U}_2^3 \rangle; \quad (25)$$

$$D_{31} = -\frac{1}{6}\langle\hat{U}_1^4\hat{U}_2^3\hat{U}_3\rangle + \frac{1}{2}\langle\hat{U}_1^4\hat{U}_2^2\hat{U}_3\hat{U}_4\rangle + \frac{2}{3}\langle\hat{U}_1^3\hat{U}_2^3\hat{U}_3\hat{U}_4\rangle + \frac{1}{2}\langle\hat{U}_1^3\hat{U}_2^2\hat{U}_3^2\hat{U}_4\rangle - 4\langle\hat{U}_1^3\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\rangle - \frac{3}{2}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3^2\hat{U}_4\hat{U}_5\rangle - \frac{1}{3}\langle\hat{U}_1^4\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\rangle + 7\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle + \frac{7}{3}\langle\hat{U}_1^3\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle - 7\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\rangle + 2\langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\hat{U}_8\rangle; \quad (26)$$

$$D_{33} = -\frac{1}{4}\langle\hat{U}_1^3\hat{U}_2^3\hat{U}_3^2\rangle + \frac{1}{4}\langle\hat{U}_1^3\hat{U}_2^3\hat{U}_3\hat{U}_4\rangle + \frac{3}{2}\langle\hat{U}_1^3\hat{U}_2^2\hat{U}_3^2\hat{U}_4\rangle - \frac{5}{2}\langle\hat{U}_1^3\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\rangle - \frac{9}{4}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3^2\hat{U}_4\hat{U}_5\rangle + \langle\hat{U}_1^3\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle + \frac{21}{4}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle - 4\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\rangle + \langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\hat{U}_8\rangle; \quad (27)$$

$$D_{46} = -\frac{1}{2}\langle\hat{U}_1^3\hat{U}_2^2\hat{U}_3^2\hat{U}_4\rangle + \langle\hat{U}_1^3\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\rangle + \frac{3}{2}\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3^2\hat{U}_4\hat{U}_5\rangle - 4\langle\hat{U}_1^2\hat{U}_2^2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle - \frac{1}{2}\langle\hat{U}_1^3\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\rangle + \frac{7}{2}\langle\hat{U}_1^2\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\rangle - \langle\hat{U}_1\hat{U}_2\hat{U}_3\hat{U}_4\hat{U}_5\hat{U}_6\hat{U}_7\hat{U}_8\rangle. \quad (28)$$

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